PUBLIC ANNOUNCEMENTS, BELIEF EXPANSION AND ABDUCTION

Hans van Ditmarsch
Angel Nepomuceno-Fernández
Logic, Language and Information Group. U. Sevilla

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Kinds of inferences I

As it is known, Peirce defined the concept of abduction, as a form of inference, though he first named the corresponding process as to formulate a hypothesis. According to him inferences could be classified as

- Inferences
  - Deductive or (analytic)
  - Synthetical
    - Induction
    - Hypothesis or abduction

Kinds of inferences II

- Hintikka has studied the Peircean notion of abduction and qualified it as the central problem of contemporary epistemology
- Both induction and abduction are sinthetical, then

  Is abduction a form of induction?

- Whewell considers a form of induction that could be taken as a precedent of abduction. Kepler is the best example of the ideal of scientific method (In the last resort, it is a diferent kind of inference)

Constructing theories

- In the process of constructing scientific theories, certain system of reasoning is adopted, which can be called the underlying logic
- Sometimes some facts arise in a way that they should have been a consequence of the corresponding postulates, but they are not, which would be surprising
- Then an epistemic action would be necessary, as extending the theory, or revising that, or modifying the logic, etc.

The four thesis

Abductive inference should accomplish:

1. *Inferential Thesis*. Abuction is, or includes, an inferential process or processes
2. *Thesis of Purpose*. The purpose of "scientific" abduction is
    1. to generate new hypotheses, and
    2. to select hypotheses for further examination
3. *Comprehension Thesis*. Scientific abduction includes *all* the operations whereby theories are engendered
4. *Autonomy Thesis*. Abduction is, or embodies, reasoning that is distinct from, and irreducible to, either deduction or induction

The link premises-conclusion

Hintikka points out that the Peirce's notion of inference has one aspect (the number 4 that has been seen above) very relevant to understand the concept of abduction: the relation between premises and conclusion. Usually rule of inference is a valid pattern of inference and may be justified in terms of such relation, either

1. The step from the premises to the conclusion is truth-preserving
2. It makes the conclusion is probable to a certain degree

But in abduction other rules or principles "of an altogether different kind" must be considered

Kinds of rules of inference

To justify an inference, Hintikka proposes two kind of rules (or principles), in keeping with the known metaphore about logic, namely

1. Definitory rules. These rules are similar to the ones that define a game like chess –deduction or scientific inquiry may be considered as a strategic game–, they tell possible moves in a given situation through the game
2. Strategic rules. These rules tell which moves are good in order to win the game

Kinds of rules of inference II

- Hintikka brings out an interrogative approach, according to which the diference between ampliative and nonampliative reasoning becames a distinction between interrogative (ampliative) and deductive (nonampliative) steps of argument
- In interrogative inquiry the thing is to anticipate the epistemic situation brought about by the answer
- All that remarks could be taken into account as an important set of accurate advice for tackling logical approaches to abduction

Logical models

Logical approaches to abduction have been proposed by several authors. One is the so called classical model of abduction or *AKM*-model (this is associated with the names of some of its more visible proponents):

- Aliseda,
- Kuipers/Kowalski, and
- Magnani/Meheus.

This logical approach is based on classical logic and it tries to define a formal framework that could explain abductive processes, where the logical parameter is pointed out.

AKM-model I

Given language L, a theory $\Theta \subseteq L$, a fact $\varphi \in L$, and a logical system $\vdash$, $(\Theta, \varphi, \vdash)$ represents an abductive problem, which may be

1. Novel abductive problem, if $\Theta \nvdash \varphi$ and $\Theta \nvdash \neg\varphi$
2. Anomalous abductive problem, if $\Theta \nvdash \varphi$ and $\Theta \vdash \neg\varphi$
    1. Given a novel abductive problem, $\psi \in L$ is a solution if $\Theta, \psi \vdash \varphi$
    2. Given an anomalous abductive problem, then
        1. performe a theory contraction to get a novel problem Θ'
        2. then solve $(\Theta', \varphi, \vdash)$

AKM-model II

The structural abduction (L. Keiff) is a variant of the *AKM*-model. Given a theory $\Theta \subseteq L$, a fact $\varphi \in L$, and a logical system (a logic) $\vdash : \mathcal{P}(L) \longmapsto L$, a new logic could be an abductive conclusion as the result of one of the inferential processes:

1. $(\Theta, \varphi, \vdash)$ is considered an abductive problem:
   - $\mathbf{\Theta} \nvdash \varphi$ and $\mathbf{\Theta} \nvdash \neg\varphi$ –anomalies can also be defined–
2. There is another logical system $\vdash^*$ such that
   1. $\vdash \subseteq \vdash^*$
   2. $\mathbf{\Theta} \vdash^* \varphi$
3. then $\vdash^*$ is the abductive solution

Belief sets I

Epistemic operation considered in belief revision are expansion, contraction and revision.

Belief expansion

Given a set of formulas $\mathcal{K}$, which can be closed under consequence $\vdash$, expansion of that by means of formula η is defined as $\mathcal{K} + \eta = \{\delta \in L : \mathcal{K}, \eta \vdash \delta\}$

Belief sets II

For $\mathcal{K}$ closed under consequence, $\mathcal{K} + \eta$ is the smallest belief set characterized by rationality postulates

1. $\mathcal{K} + \eta$ is a belief set — type
2. $\eta \in \mathcal{K} + \eta$ — success
3. $\mathcal{K} \subseteq \mathcal{K} + \eta$ — expansion
4. If $\eta \in \mathcal{K}$, then $\mathcal{K} + \eta = \mathcal{K}$ — minimal action
5. If $\mathcal{K} \subseteq \mathcal{K}'$, then $\mathcal{K} + \eta \subseteq \mathcal{K} + \eta$ — monotony

Abduction and expansion

Abductive expansion

Given an abductive problem $(\Theta, \varphi, \vdash)$, the abductive expansion of Θ with respect to φ (and $\vdash$) is defined

$$Abdex_{\varphi}(\Theta) = \Theta \cup \{\chi \in L : \Theta, \chi \vdash \varphi\}$$

Abductive expansion II

Theorem 1

Let $(\Theta, \varphi \vdash)$ be an abductive problem, and the set

$$\Delta_{\Theta,\varphi} = \bigcup_{\chi \in Abdex_{\varphi}(\Theta)} (\Theta + \chi)$$

Then $Abdex_{\varphi}(\Theta) = \Delta_{\Theta,\varphi}$

Abductive expansion III

Schematic proof:

1. Suppose $\eta \in \Delta_\Theta$, then $\exists \psi_k$ such that $\eta \in \Theta + \psi_k$ because of which $\Theta, \psi_k \vdash \eta$, since $\Theta, \psi_k \vdash \varphi$, we have that $\Theta, \psi_k \wedge \eta \vdash \varphi$. So $\psi_k \wedge \eta \notin \Theta$ and $\psi_k \notin \Theta$ (in other case, $\Theta \vdash \varphi$, but it is contradictory with the fact that $(\Theta, \varphi \vdash)$ is an abductive problem). Then two cases are possible:
    1. $\eta \in \Theta$. Then $\eta \in Abdex_\varphi(\Theta)$
    2. $\eta \notin \Theta$. Then $\eta \in \{\chi \in L : \Theta, \chi \vdash \varphi\}$, so that $\Theta, \eta \vdash \varphi$, then $\eta \in Abdex_\varphi(\Theta)$
2. And reciprocally

Abductive revision

Let $(\Theta, \varphi, \vdash)$ be an abductive problem with φ as anomaly. Then $\Theta \nvdash \varphi$ and $\Theta \vdash \neg\varphi$.

Abductive contraction

$Abdcon_{\neg\varphi}(\Theta) = \Theta - \{\chi \in \Theta : \Theta \vdash \neg\varphi\} = \Theta' = \Theta - \{\chi_1, ..., \chi_k\}$, where Θ' is the minimal set such that $\Theta' \nvdash \neg\varphi$

Then it can be defined

Abductive revision

$Abdre_{\varphi}(\Theta) = Abdex_{\varphi}(Abdcon_{\neg\varphi}(\Theta)) = Abdcon_{\neg\varphi}(\Theta) + \varphi$

So revision is a process of contraction and expansion

KD45 as underyling logic

The language L, for propositonal variables $\mathcal{P}$ and agents $\mathcal{A}$

$$\varphi := p \mid \neg\psi \mid \psi \rightarrow \chi \mid B_a\psi$$

The system $KD45$ consists of all propositional tautologies and

1. $B_a(\psi \rightarrow \chi) \rightarrow (B_a\psi \rightarrow B_a\chi)$
2. $\neg B_a\bot$ [or, equivalently, $B_a\psi \rightarrow \neg B_a\neg\psi$]
3. $B_a\psi \rightarrow B_aB_a\psi$
4. $\neg B_a\psi \rightarrow B_a\neg B_a\psi$
5. Rules: *modus ponens* and necessitation:

$$\frac{\vdash \psi}{\vdash B_a\psi}$$

Forms of closure

Given $\Theta \subset L$, $Cn_{KD45}(\Theta) = \{\chi \in L : \Theta \vdash_{KD45} \chi\}$ –to abbreviate, Cn instead of Cn_{KD45} and $\vdash$ instead of $\vdash_{KD45}$–.

1. Θ is closed under Cn iff

$$\Theta = Cn(\Theta)$$

2. Θ consistent is closed under belief iff for all $\chi \in L$ and $a \in \mathcal{A}$,

$$\chi \in \Theta \text{ iff } B_a\chi \in \Theta$$

Examples I

$\Theta = \{B_a(\alpha \rightarrow \beta)\}$ (Θ is not closed under belief). Abductive problem: $(\Theta, \beta, \vdash)$, with β as novelty. Then

- $\Theta \nvdash B_a\beta$ and $\Theta \nvdash \neg B_a\beta$
- Since $\Theta, \alpha \nvdash \bot$, $\Theta + B_a\alpha$ contains abductive solutions:

$$B_a\alpha, B_a\beta \in \Theta + B_a\alpha,$$

though

$$B_aB_a\alpha, B_aB_a\beta \in \Theta + B_a\alpha \text{ and so on}$$

Examples II

Θ is closed under belief, $B_a(\alpha \rightarrow \beta) \in \Theta$, abductive problem $(\Theta, \beta, \vdash)$ and $\beta \notin \Theta$. Then

- $\neg B_a\beta \in \Theta$ and $B_a\neg B_a\beta \in \Theta$
- $\Theta + B_a\beta$ is not consistent:

$$\neg B_a\beta \in \Theta + B_a\beta \text{ and } B_a\beta \in \Theta + B_a\beta$$

Examples III

Steps to solve the abductive problems

1. Abductive constraction $Abdcon_{\neg B_a\beta}(\Theta) = \Theta'$
2. Abductive expansion $Abdex_{B_a\beta}(\Theta') = \Delta_{\Theta', B_a\beta}$
3. By combining 1 and 2 (revision):

$$Abdre_{B_a\beta}(\Theta) = Abdex_{B_a\beta}(Abdcon_{\neg B_a\beta}(\Theta))$$

New operators

L^* is defined by the BNF:

$$\varphi ::= p \mid \neg\psi \mid \psi \rightarrow \chi \mid B_a\psi \mid [\oplus\chi]\psi \mid [\ominus\chi]\psi \mid [\circledast\chi]\psi$$

where operators should be read as

1. $[\oplus\chi]\psi$: after expansion with χ, ψ holds
2. $[\ominus\chi]\psi$: after contraction with χ, ψ holds
3. $[\circledast\chi]\psi$: after revision with χ, ψ holds

(for semantics, take into account models with respect to χ)

Examples IV

1. Abductive problem: $(\Theta, \beta, \vdash)$. If $[\oplus\alpha]\beta \in \Theta$, then
   - The theory provides us with an explanation: $\Theta \vdash \beta$,

$$\text{since } \Theta + \alpha \subset \mathit{Abdex}_\beta(\Theta),\ \{[\oplus\alpha]\beta\} \vdash \beta$$

2. The former example: abductive problem $(\Theta, \beta, \vdash)$, with $B_a(\alpha \to \beta) \in \Theta$, $\beta \notin \Theta$, Θ closed under belief: $\neg B_a\beta \in \Theta$, then
   1. Take $\Theta' = \Theta \cup \{[\circledast\neg B_a\beta]\beta\}$, then
   2. $\Theta' \vdash \beta$

Concluding remarks

- For (consistent) theories that are closed under deductive consequence, abductive expansion is not possible, since such theories cannot increase
- For theories that contain the mentioned epistemic operators, the theory can be explain the fact (which would not be so surprising)

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MOITO OBRIGADO!

THANK YOU VERY MUCH!